Determination of the off-diagonal element of the dielectric tensor without measuring the ellipticity

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In addition to the measurement of the Kerr rotation angle, the measurement of the ellipticity is generally known to be necessary for determining the off-diagonal element of the dielectric tensor. We have found a new method for determining the off-diagonal element of the dielectric tensor, without measuring the ellipticity, for a sample deposited on a transparent substrate. © *1996 American Institute of Physics.* [S0003-6951(96)00420-2]

Magneto-optical (MO) Kerr effects have had much attention over the last decade, primarily due to the application of the effects to erasable high-density data storage. MO Kerr effects are the change of the polarization state of a reflected light from a magnetized sample. When a linearly polarized light is reflected from a magnetic material, the reflected light is generally elliptically polarized. The rotation angle of the ellipse is called the Kerr rotation angle, and the arctangent of the ratio of its short and long axis is the Kerr ellipticity. When the direction of the magnetization is perpendicular to the film plane, it is called the polar Kerr effect. It is caused by the off-diagonal element of the dielectric tensor, which is a fundamental response function of a magnetized sample to the external electric field.[1] Therefore, determination of the off-diagonal element is important for understanding MO effects and also, for developing new MO recording materials.

It is generally known that the ellipticity as well as the Kerr rotation angle and the complex refractive index $n+ik$ should be measured to determine the off-diagonal element.[1,2] However, most experimental methods, except the phase modulation method adopting a photoelastic modulator, should employ a λ/4 plate or a Soleil–Babinet compensator corresponding to a given wavelength in measuring the ellipticity.[2–5] Therefore, those methods cause much inconvenience to carry out the spectrometric measurements.

In this letter, we report a new method to determine the off-diagonal element of the dielectric tensor without measuring the ellipticity of a magnetized film. This method could be applied to any magnetic materials prepared on a transparent substrate with a known refractive index. We present an analytic solution for bulk and a numerical solution for thin film.

When a sample having cubic symmetry or higher symmetry is magnetized in the z direction, using symmetry operations and Onsager relations[6] the dielectric tensor of the sample is given by:

$$\tilde{\epsilon}=\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ -\epsilon_{xy} & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}. \qquad (1)$$

The elements of the tensor, ϵ_{xx} and ϵ_{xy}, are generally complex numbers. The left- and right-circular polarization vectors are the basis vectors of the normal incidence light to the sample of the dielectric tensor $\tilde{\epsilon}$. Therefore, the complex refractive indices are given by $n_\pm^2=\epsilon_{xx}\pm i\epsilon_{xy}$ for each circular polarization in the optical wavelength region.[7] Then, considering only the first-order approximation of $\epsilon_{xy}/\epsilon_{xx}$, one can get a well-known relation as follows:[1,2,6]

$$\theta_K+i\varepsilon_K=\frac{n_0\epsilon_{xy}}{\sqrt{\epsilon_{xx}}(n_0^2-\epsilon_{xx})}, \qquad (2)$$

where θ_K,ε_K, and n_0 are the Kerr rotation angle, the Kerr ellipticity, and the refractive index of surrounding medium, respectively. Putting $\epsilon_{xx}=(n+ik)^2$ and $\epsilon_{xy}=\epsilon'_{xy}+i\epsilon''_{xy}$ in Eq. (2), θ_K and ε_K can be expressed in a matrix form as follows:

$$\begin{pmatrix} \theta_K \\ \varepsilon_K \end{pmatrix}=\begin{pmatrix} A & B \\ -B & A \end{pmatrix}\begin{pmatrix} \epsilon'_{xy} \\ \epsilon''_{xy} \end{pmatrix}, \qquad (3)$$

where A and B are given by

$$A=\frac{n_0n(n_0^2-n^2+3k^2)}{(n^2+k^2)[(n_0^2-n^2-k^2)^2+4n_0^2k^2]}, \qquad (4)$$

$$B=\frac{n_0k(n_0^2-3n^2+k^2)}{(n^2+k^2)[(n_0^2-n^2-k^2)^2+4n_0^2k^2]}. \qquad (5)$$

Therefore, if the A,B,θ_K, and ε_K are known, ϵ_{xy} can be obtained easily.[1,2,6]

In Eqs. (3), (4), and (5), it should be pointed out that θ_K and ε_K are dependent on the refractive index of surrounding medium, n_0. So if the light entered through a transparent substrate, n_0 would be replaced by the refractive index of the substrate n_s. In this case, one may ignore the multiple interference effect on the Kerr rotation angle for a light source having short coherence length. Because the Faraday effect of the substrate is proportional to an external magnetic field, its effect in the measurement of the Kerr rotation angle for the substrate incidence can be easily subtracted. Therefore, a similar relation as Eq. (3) is obtained for the substrate incidence:

$$\begin{pmatrix} \theta_K^s \\ \varepsilon_K^s \end{pmatrix}=\begin{pmatrix} A' & B' \\ -B' & A' \end{pmatrix}\begin{pmatrix} \epsilon'_{xy} \\ \epsilon''_{xy} \end{pmatrix}, \qquad (3')$$

where θ_K^s and ε_K^s are the Kerr rotation angle and the ellipticity for the substrate incidence, and A' and B' are the same relation as A and B except that n_0 is replaced with n_s.

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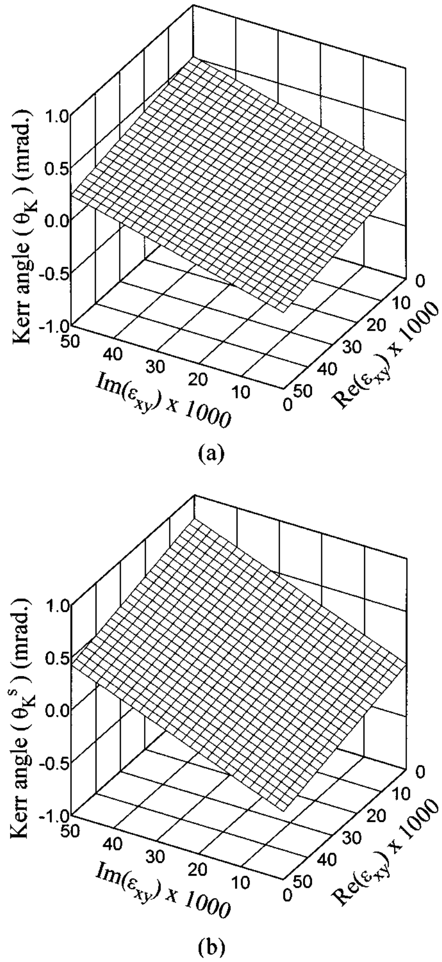

FIG. 1. Dependence of the Kerr rotation of a bulk sample on the real and the imaginary parts of the off-diagonal element of the dielectric tensor (a) for the film incidence and (b) for the substrate incidence.

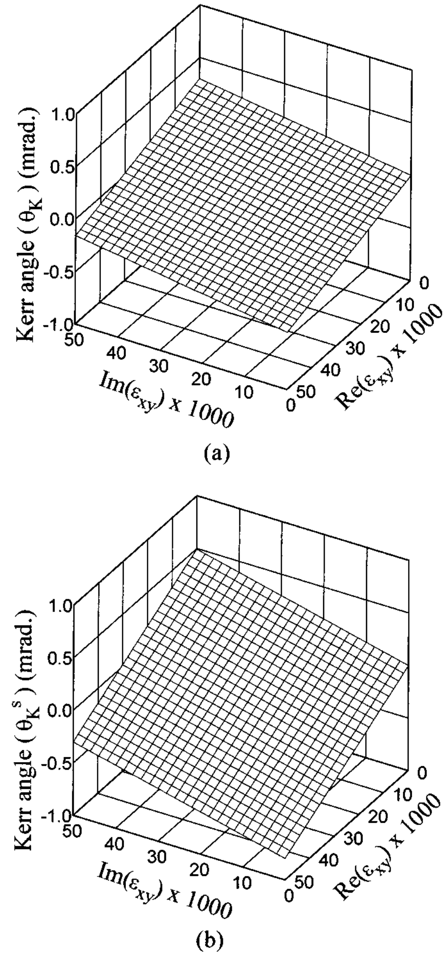

FIG. 2. Dependence of the Kerr rotation of a 300-Å thin film on the real and the imaginary parts of the off-diagonal element of the dielectric tensor (a) for the thin film incidence and (b) for the substrate incidence.

From Eqs. (3) and (3′), one may easily relate θ_K and θ_K^s to ϵ'_{xy} and ϵ''_{xy} as follows:

$$\begin{pmatrix} \theta_K \\ \theta_K^s \end{pmatrix} = N \cdot \begin{pmatrix} \epsilon'_{xy} \\ \epsilon''_{xy} \end{pmatrix}, \qquad (6)$$

where N is given by

$$N = \begin{pmatrix} A & B \\ A' & B' \end{pmatrix}. \qquad (7)$$

The relations of θ_K and θ_K^s vs ϵ'_{xy} and ϵ''_{xy} are depicted by a plane as shown in Figs. 1(a) and 1(b) for a given value of $\epsilon_{xx} = -3.0527 + 19.2864i$ for Fe film and $n_s = 1.5$ for the glass substrate at the wavelength 6328 Å, respectively. Since the secular equation of the matrix N is given by

$$\det|N| = AB' - BA' \propto n_s^2 - n_0^2 \ , \qquad (8)$$

an inverse matrix of N always exists if n_s is different from n_0. The condition of $n_s \neq n_0$ is satisfied for most substrates in visible-wavelength region. Hence ϵ'_{xy} and ϵ''_{xy} can be obtained from

$$\begin{pmatrix} \epsilon'_{xy} \\ \epsilon''_{xy} \end{pmatrix} = N^{-1} \begin{pmatrix} \theta_K \\ \theta_K^s \end{pmatrix}. \qquad (9)$$

Therefore, one can analytically obtain ϵ'_{xy} and ϵ''_{xy} of the thick sample prepared on a transparent substrate from the measurements of θ_K and θ_K^s. The ellipticity can also be obtained from the relation as follows:

$$\begin{pmatrix} \theta_K \\ \varepsilon_K \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \dfrac{AA' + BB'}{A'B - AB'} & \dfrac{A^2 + B^2}{AB' - A'B} \end{pmatrix} \begin{pmatrix} \theta_K \\ \theta_K^s \end{pmatrix}. \qquad (10)$$

So far, we have considered an optically thick film where the multiple interference could be ignored. Therefore, the previous results are not applicable to a thin film where the multiple interference should be taken into account. In this situation, using the medium propagation matrix method presented by Zak *et al.*,[8] θ_K and ε_K might be expressed by a functional formula as follows:

$$\theta_K + i\varepsilon_K = f(\tilde{n}_i, \epsilon_{xy}, d), \qquad (11)$$

where d is the thickness of a film, $\tilde{n}_i = n_i + ik_i$ represents the complex refractive index of each layer of substrate, film, and air at a given wavelength. Hence, θ_K is given by the real part of the function f. Considering that MO effect is the first-order effect of ϵ_{xy} for a given $\tilde{n}_i$ even in the presence of the multiple reflections, the Kerr rotation angles of θ_K and θ_K^s can be expressed by

$$\theta_K = A(\tilde{n}_i, d)\epsilon'_{xy} + B(\tilde{n}_i, d)\epsilon''_{xy}, \quad (12)$$

$$\theta_K^s = A^s(\tilde{n}_i, d)\epsilon'_{xy} + B^s(\tilde{n}_i, d)\epsilon''_{xy} \ . \quad (13)$$

The coefficients of A, B, A^s, and B^s are generally very complicated functions for a thin film, but they can be numerically determined once $\tilde{n}$ and d are provided. Therefore, using the measured values of θ_K and θ_K^s the off-diagonal element of ϵ'_{xy} and ϵ''_{xy} can be obtained by solving two independent linear equations.

To check validity and independence of Eqs. (12) and (13), the magneto-optical response of 300-Å Fe on a glass substrate was calculated as an example. Using the value of $\tilde{n} = 2.87 + 3.36i$ for Fe film and $n_s = 1.5$ for the glass substrate at the wavelength 6328 Å, we have obtained θ_K and θ_K^s as plotted in Figs. 2(a) and 2(b), respectively. It can be seen in these figures that θ_K and θ_K^s are represented as planes in ϵ'_{xy} - ϵ''_{xy} coordinates which proves validity of the linear relationship in Eqs. (12) and (13). While, the fact that two planes in Figs. 2(a) and 2(b) have different normal vectors implies that Eqs. (12) and (13) are independent.

Once ϵ'_{xy} and ϵ''_{xy} are determined by the present method, the ellipticity ε_K is easily obtained using Eq. (11).[8] With determined values of θ_K and ε_K, one may then calculate ϵ'_{xy} and ϵ''_{xy} by employing the Newton–Raphson's method.[9] We have confirmed that the results were exactly same as the values of ϵ'_{xy} and ϵ''_{xy} determined by the present method.

It should be mentioned that in principle there is no problem to apply the present method to an absorbing substrate once the complex refractive index of the substrate is known. However, the signal-to-noise ratio is expected to be poor for a high absorbing substrate.

In conclusion, we have developed a new method for determining the off-diagonal element of the dielectric tensor without measuring the ellipticity. Two Kerr rotation angles measured from the film side and the substrate side are used in the present method, where ϵ_{xy} is determined analytically for a bulk sample, while they are determined numerically for a thin film. The present method is always applicable once the refractive index and thickness of a sample and the refractive index of a substrate are known, providing that $n_s \neq n_0$.

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